

International Journal of Mass Spectrometry 185/186/187 (1999) 97–105

Theoretical prediction of charged droplet evaporation and fission in electrospray ionization

Keqi Tang¹, Richard D. Smith*

Environmental Molecular Science Laboratory, Mail Stop K8-98, Pacific Northwest National Laboratory, P.O. Box 999, Richland, WA 99352, USA

Received 6 May 1998; accepted 19 July 1998

Abstract

A theoretical analysis was performed on the droplet asymmetric fission in electrospray ionization process. The formulated model is able to predict the diameter, charge and the percentage of charge at the Rayleigh charge limit for both progeny droplets and residue parent droplet during each fission process based upon possible different numbers of progeny droplets generated by the fission event. The evaporation time required for both the progeny droplets and the residue parent droplet to become unstable after each fission process was also calculated and compared. The results of these calculations showed that the evaporation time for the progeny droplets is more than an order of magnitude less than that of the residue parent droplet, indicating that the progeny droplets are the primary ion source for conventional electrospray ionization sources. (Int J Mass Spectrom 185/186/187 (1999) 97–105) © 1999 Elsevier Science B.V.

Keywords: Electrospray ionization; Theory; Droplet fission; Rayleigh limit; Droplets

1. Introduction

Electrosprays for the generation of highly charged, and ideally monodisperse, droplets have been used in many areas, as reviewed by Bailey [1]. The most important application presently is in electrospray ionization (ESI), that has proven broadly effective for dispersing ions in the liquid phase into the gas phase. Within about a decade since its combination with mass spectrometry by Yamashita and Fenn [2], ESI

has become the dominant ion source for many mass spectrometric applications. Its capability for generating multiply charged gas phase ions has enabled large biomolecules to be studied using mass spectrometer with limited *m/z* range [3].

A key question relevant to ESI is the origin of the ions "created." A better understanding of the mechanism for ion generation in ESI will help to explain many currently unresolved issues and allow more effective use of ESI sources for mass spectrometry (ESI-MS) [4] and the development of advanced ion sources, such as those employing "ion funnel" technology [5]. Currently existing mechanisms for the ion formation in ESI fall largely into two groups, namely, the charge residue model (CRM) and the field-assisted ion desorption model (FDM). CRM, as proposed

^{*} Corresponding author. E-mail: rd_smith@pnl.gov

¹ Current address: Finnigan Corporation, 355 River Oaks Parkway, San Jose, CA 95134.

Dedicated to Professor Michael T. Bowers on the occasion of his 60th birthday.

originally by Dole and co-workers [6], suggests that ions are formed from the charged residue remaining after the completion of evaporation of the original droplets. On the other hand, FDM, first proposed by Iribarne and Thomson [7] for the small ion formation from liquid droplets, suggests that ions be formed by the direct field desorption from the droplet surface when the electric field on the droplet surface reaches a critical value, on the order of 10^9 V/m. Both models are subject to continued debate and comparison with experiment, and while some refinements have been proposed, no resolution of this issue is broadly accepted [8]. We have previously suggested that a variation of the CRM appears to be able to explain most of the experimental results [4].

A common process, implied or explicitly stated by all the ion formation models as the first step of the ion formation in electrospray, is that the droplet coulombic fission that occurs when the charge on the droplet reaches the Rayleigh charge limit [9], corresponding to a critical condition when electric force overcomes the liquid surface tension for a spherical droplet. A better understanding of this droplet fission process is thus crucial for correctly predicting the mechanism of ESI formation.

Interesting in the droplet fission process has preceded before the application of electrospray to mass spectrometry because of its implications to many different areas ranging from atmospheric physics [10] to combustion [11]. Both mass and charge loss during the fission process were measured by several investigators using different techniques [12–14]. Among them, Taflin et al. [14] presented the most accurate measurements to date obtained by using an optical resonance method to accurately measure droplet size variation during droplet evaporation-fission process. Their results showed a mass loss of 1–2.3% and a charge loss of 10–18% during each droplet fission process. A more recent investigation, reported by Gomez and Tang [15], revealed photographically the droplet fission process in the electrospray. Significant droplet distortion was observed before each droplet fission process, which tended to generate random numbers of progeny droplets. A theoretical calculation was also performed on the cascade evaporationfission process for each generation of droplets in the electrospray. To simplify the calculation, a fixed diameter ratio of progeny droplet to parent droplet was assumed in their analysis. A similar assumption was also used by Kebarle and Tang [16], under which a fixed number of progeny droplets for each fission process was implied.

To generalize the theoretical analysis of the droplet fission process, we present here a theoretical model in which the number of progeny droplets generated from each fission process was assumed to be a random variable. The model allows us to predict both charge and size for each possible group of droplets before and after each specific fission process and the evaporation time required for each group of droplets to subsequently become unstable as a function of the number of progeny droplets generated. Several new implications based on these results are described.

2. Theoretical model

A complete theoretical model can be formulated including evaporation and coulombic fission in the electrospray ionization process, if the following assumptions are used, based on the experimental measurements and observations from others:

(1) A charged droplet on average loses 2% of its mass and 15% of its charge to the progeny droplets in each fission process [14].

(2) Each fission process will tend to generate a specific number and size for progeny droplets; i.e. progeny droplets are assumed to be monodisperse and carry an equal amount of charge [15].

Mathematically, the above assumptions can be specifically expressed as

$$
m_{dT} = 0.02m_0; \t q_{dT} = 0.15q_0 \t\t(1)
$$

and

$$
m_{dT} = n m_d; \qquad q_{dT} = n q_d \tag{2}
$$

where m_0 and q_0 are the mass and charge of the parent droplet before fission; m_{dT} and q_{dT} are the total progeny mass and charge ejected by the parent droplet in each fission process; m_d and q_d are the average mass and charge for each progeny droplet and *n* is the number of progeny droplets generated by each fission process.

From Eqs. (1) and (2), we have

$$
\frac{q_d}{m_d} = \frac{0.15q_0}{0.02m_0} \tag{3}
$$

Because $m_d = \frac{1}{6} \pi \rho d_d^3$ and $m_0 = \frac{1}{6} \pi \rho d_0^3$, where d_d and d_0 are the average diameter of the progeny droplets and the diameter of parent droplet before fission, respectively, and ρ is the density of the droplet, we then have

$$
\frac{q_d}{d_d^3} = 7.5 \frac{q_0}{d_0^3} \quad \text{or} \quad \frac{q_d}{d_d^2} = 7.5 \frac{d_d}{d_0} \times \frac{q_0}{d_0^2} \tag{4}
$$

From mass conservation and assumption (1), we have

$$
\frac{nd_d^3}{d_0^3} = 0.02 \quad \text{or} \quad \frac{d_d}{d_0} = \left(\frac{0.02}{n}\right)^{1/3} \tag{5}
$$

By substituting Eq. (5) into Eq. (4), we obtain

$$
\frac{q_d}{d_d^2} = 7.5 \left(\frac{0.02}{n}\right)^{1/3} \frac{q_0}{d_0^2} = 2.036 n^{-(1/3)} \frac{q_0}{d_0^2} \tag{6}
$$

Because Rayleigh charge limit for both the parent droplet and the progeny droplets can be expressed as $q_{0r} = (8\pi^2 \varepsilon_0 \gamma)^{1/2} d_0^{3/2}$ and $q_{dr} = (8\pi^2 \varepsilon_0 \gamma)^{1/2} d_d^{3/2}$, respectively, where ε_0 is the electric permitivity of vacuum and γ is the surface tension of the liquid, we have

$$
q_{dr} = \left(\frac{d_d}{d_0}\right)^{3/2} q_{0r} \quad \text{or} \quad \frac{q_{dr}}{d_d^2} = \left(\frac{d_0}{d_d}\right)^{1/2} \frac{q_{0r}}{d_0^2} \tag{7}
$$

By taking the ratio of Eqs. (6) and (7), it becomes

$$
\frac{q_d}{q_{dr}} = 2.036n^{-(1/3)} \left(\frac{d_d}{d_0}\right)^{1/2} \frac{q_0}{q_{0r}}\tag{8}
$$

By substituting Eq. (5) into (8), we obtain

$$
\frac{q_d}{q_{dr}} = 2.036n^{-(1/3)} \left(\frac{0.02}{n}\right)^{1/6} \frac{q_0}{q_{0r}} = 1.06n^{-(1/2)} \frac{q_0}{q_{0r}} \tag{9}
$$

Combining Eqs. (5), (6), and (9), the droplet size, surface charge density and percentage of droplet charge at the Rayleigh charge limit can all be predicted for progeny droplets for each fission process. It is clearly indicated by Eqs. (5), (6), and (9) that the progeny droplets can have different sizes, surface charge densities and percentage of the Rayleigh charge limit at the time of formation, depending upon the number of progeny droplets generated in each fission process. The results and implications of these calculations will be discussed later.

After the initial droplet fission process, both the "residue" parent droplet and the progeny droplets will evaporate as they move further "downstream" in the electrospray plume (i.e. drift in the electric field used to create the electrospray). (Both the ambient temperature and solvent vapor pressure are in reality somewhat variable, depending upon the detailed design and arrangement of the ESI source, and impact the rate of solvent evaporation from the droplets. We do not explicitly consider the complications introduced by these considerations in this work.)

Because it has been observed experimentally that charged droplet retains all the charge on the surface during evaporation [13], the surface charge density for both the residue parent droplet and the progeny droplets will continuously increase during evaporation of the neutral solvent. Eventually, they will become unstable again as they approach a certain percentage of the Rayleigh charge limit. To investigate the evaporation process for both types of droplets and predict the times required for both residue parent and progeny droplets to become unstable, in the present analysis the evaporation theory for a isolated droplet in a diffusion-controlled regime under constant environmental conditions was used [17]. Accordingly, the variation of the droplet diameter during the evaporation can be expressed as

$$
d_d^2 - d_d^{*2} = kt \tag{10}
$$

for progeny droplet, where d_d^* is the diameter of the progeny droplet at any time *t*, and *k* is the evaporation rate constant. At any instant *t* during evaporation, the droplet charge as a percentage of the Rayleigh charge limit can be expressed as

$$
\frac{q_{d}^*}{q_{dr}^*} = \frac{q_d}{q_{dr}^*} \tag{11}
$$

where $q_{dr}^* = (8\pi^2 \varepsilon_0 \gamma)^{1/2} d_d^{*3/2}$ is the Rayleigh limit for progeny droplet at time *t*. Similar to Eq. (7), we have

$$
q_{dr}^{*} = \left(\frac{d_{d}^{*}}{d_{d}}\right)^{3/2} q_{dr}
$$
 (12)

By substituting Eq. (12) into (11) and making use of Eq. (9), we have

$$
\frac{q_d^*}{q_{dr}^*} = \left(\frac{d_d}{d_d^*}\right)^{3/2} \frac{q_d}{q_{dr}} = 1.06n^{-(1/2)} \left(\frac{d_d}{d_d^*}\right)^{3/2} \frac{q_0}{q_{0r}} \tag{13}
$$

If we assume progeny droplets become unstable at the same percentage of Rayleigh charge limit as the parent droplet, then the progeny droplet will be unstable when

$$
1.06n^{-(1/2)}\left(\frac{d_d}{d_d^*}\right)^{3/2} = 1\tag{14}
$$

according to Eq. (13). By substituting Eq. (10) into Eq. (14), we have

$$
1.06n^{-(1/2)} \left(\frac{d_d}{(d_d^2 - kt_d)^{1/2}} \right)^{3/2} = 1 \tag{15}
$$

where t_d is the time required for the progeny droplet to become unstable. Through a simple manipulation to Eq. (15), one readily obtains

$$
1.06^{4/3}n^{-(2/3)}d_d^2 = d_d^2 - kt_d
$$

or

$$
t_d = \frac{d_d^2}{k} (1 - 1.06^{4/3}n^{-(2/3)})
$$
 (16)

By substituting Eq. (5) into Eq. (16) , we have

$$
t_d = \frac{d_0^2}{k} \left(\frac{0.02}{n}\right)^{2/3} (1 - 1.06^{4/3} n^{-(2/3)}) \tag{17}
$$

For the residue parent droplet with diameter d_p and charge q_p , evaporation will also make it unstable again at later time t_p . Similar to the progeny droplets, we will have

$$
\frac{q_p^*}{q_{pr}^*} = \frac{q_p}{q_{pr}^*} \tag{18}
$$

where q_p^* and q_{pr}^* are the charge and Rayleigh charge limit for the residue parent droplet at any instant during its evaporation. Since, similar to Eq. (12), q_{nr}^* $= (d_p^*/d_p)^{3/2}q_{pr}$, where d_p^* is the diameter of the residue parent droplet, then

$$
\frac{q_{p}^{*}}{q_{pr}^{*}} = \left(\frac{d_{p}}{d_{p}^{*}}\right)^{3/2} \frac{q_{p}}{q_{pr}}
$$
\n(19)

Because we also have $q_p = 0.85q_0$, from assumption 1, and $q_{pr} = (d_p/d_0)^{3/2} q_{0r}$, thus

$$
\frac{q_{p}^{*}}{q_{pr}^{*}} = 0.85 \left(\frac{d_{p}}{d_{p}^{*}}\right)^{3/2} \frac{q_{0}}{q_{pr}}
$$

$$
= 0.85 \left(\frac{d_{p}}{d_{p}^{*}}\right)^{3/2} \left(\frac{d_{0}}{d_{p}}\right)^{3/2} \frac{q_{0}}{q_{0r}}
$$

$$
= 0.85 \left(\frac{d_{0}}{d_{p}^{*}}\right)^{3/2} \frac{q_{0}}{q_{0r}}
$$
(20)

If we again assume the residue parent droplet becomes unstable at the same percentage of Rayleigh Charge limit as the initial parent droplet, then

$$
0.85 \left(\frac{d_0}{d_p^*}\right)^{3/2} = 1\tag{21}
$$

Because $m_p = 0.98m_0$ or $d_p^3 = 0.98d_0^3$, from assumption (1), and $d_p^2 - d_p^{*2} = kt_p$, where t_p is the time required for the residue parent droplet to become unstable, we then have

$$
0.85^{4/3}d_0^2 = 0.98^{2/3}d_0^2 - kt_p \tag{22}
$$

and

$$
t_p = \frac{d_0^2}{k} (0.98^{2/3} - 0.85^{4/3}) = 0.181 \frac{d_0^2}{k}
$$
 (23)

Eqs. (17) and (23) can be used to predict and compare the time for both the progeny droplets and the residue parent droplet to become unstable after the initial droplet fission event. The model formulated allows us to calculate size, charge and percentage of Rayleigh charge limit for both the progeny droplets

Number of Progeny Droplets in Each Fission

Fig. 1. Diameter ratio of progeny droplet to parent droplet before fission vs. number of progeny droplets generated in the fission process.

and residue parent droplet right after the initial droplet fission and the time required to bring both types of the droplets to instability (fission) again due to droplet evaporation. Theoretically, each set of droplets from each subsequent fission can be followed until (after final evaporation of solvent) individual multiply charged ions are eventually generated in the electrospray.

3. Results and discussion

Fig. 1 shows the calculated size ratio of the progeny droplet to the initial parent droplet during each fission process as a function of the number of progeny droplets, obtained using Eq. (5). These results indicate clearly that different sizes of progeny droplets can be generated by each fission process depending the number of progeny droplets being generated. It has been observed experimentally [15] that the fission process tends to generate random numbers of progeny droplets. This appears to be determined largely by the deformation of the parent droplets in the fission process, which is affected by both the local charge density on the droplet surface and the electric field in the vicinity of the fission droplet induced mostly by the spacing charge field. It has been also observed that droplet fission process is asymmetric and takes the similar cone-jet morphology as the formation of stable monodisperse electrospray [18]. Consequently, the assumption (2) used in the current analysis can be justified.

Fig. 2 compares the surface charge density of the progeny droplets to those of both the parent droplet just before the asymmetric fission and the residue of the parent droplet after the asymmetric fission event, according to the number of progeny droplets. These calculations indicate that the surface charge density of the progeny droplets can be either higher or lower than those for both types of parent droplets, depending on the number of progeny droplets generated in

Fig. 2. Surface charge density ratio of progeny droplet to parent droplet just before fission (left *y* axis) and residue parent droplet after fission (right y axis) vs. number of progeny droplets generated in fission process, where d_d and q_d are the diameter and the charge of the progeny droplet; d_0 and q_0 are the diameter and the charge of the parent droplet just before fission; d_p and q_p are the diameter and the charge of the residue parent droplet after fission.

each fission process. Fig. 3 further compares the percentage of droplet charge at the Rayleigh charge limit for the progeny droplets immediately following the fission event. As is indicated by Fig. 3, the ratio of the progeny droplet charge to the Rayleigh charge limit after the fission process will be generally smaller than that for the initial parent droplet just before fission, except for the special case, where only one progeny droplet is generated by the fission process. In this case, as predicted by the theoretical model, the progeny droplet will be immediately unstable after the fission process. We consider this case unlikely due to the cone-jet morphology of the asymmetric fission, and it will not be considered further. (It can also be calculated from the current model that the ratio of the residue parent droplet charge to the Rayleigh charge limit is 85% of the ratio for the initial parent droplet.) It is thus concluded that both the progeny droplets and the residue parent droplet are stable immediately after the fission event.

As droplet evaporation proceeds after the fission event, the surface charge density will continuously increase on both progeny droplets and the residue parent droplet until they become unstable again. Fig. 4 shows the calculated times for progeny droplets to become unstable as a function of different possible number of progeny droplets in an event, obtained by assuming the next event occurs when the percentage of the droplet charge to the Rayleigh charge limit reaches the same value as for the initial parent droplet. By comparing the results shown in Fig. 4 to Eq. (23), one sees that the progeny droplets become unstable much faster (by more than an order of magnitude faster) than the residue parent droplet. This comparison together with the fact that the progeny droplets, as indicated by Fig. 1, are much smaller than the parent

Fig. 3. Ratio of percentage of progeny droplet charge to Rayleigh charge limit to that of parent droplet just before fission vs. number of progeny droplets generated in fission process.

droplet indicates that the progeny droplets will be most likely the primary source of ions for a conventional electrospray ionization source. The primary role of the residue parent droplet will be to generate additional progeny droplets through the subsequent fission process.

Thus, while the progeny droplets, and their progeny, progress through this cascade of fission events, the residue from the original parent droplet will be much larger. The average number of fission events that can occur would obviously depend upon the initial droplet size. In an ESI source it is generally advantageous to speed the ion formation process since a slow droplet evaporation process allows time for greater coulombic expansion, dilution of ion density of the volume "sampled" by the mass spectrometer, and adversely impacts sensitivity. Thus, ion sources are generally heated and the heating necessary for optimum sensitivity is somewhat solution dependent. We believe that this is also one of the reasons why studies of noncovalent complexes generally yield significant lower sensitivities even though the same amount of material is dispersed by the ESI process as for the much more sensitive measurements under normal (higher temperature) source conditions. For the very gentle heating conditions generally used to observe noncovalent complexes one only detects ions formed from the smallest progeny droplets in the earliest stages of the ESI process. A challenge for future ion source designs will be to realize conditions where the ion formation process has sufficient time to generate ions with optimum efficiency, without imparting excessive energy in ions formed by early events and which also avoids the coulombically driven "dilution" of the ions. In this context, the electrodynamic "ion funnel" technology recently de-

Number of Progeny Droplets in Each Fission

Fig. 4. Evaporation time required for progeny droplet to become unstable vs. number of progeny droplets generated in the fission process.

scribed by us for reversing space charge expansion in the interface [5], may prove advantageous.

As originally observed by Gomez and Tang [15], the progeny droplets were mostly ejected in the sidewise direction toward the periphery region of the electrospray. This phenomenon, together with the fact that the progeny droplets will be pushed towards the edge of the electrospray due to its smaller inertia (similar to the separation of satellite droplets and the larger primary droplets observed in the spray formation process [18]), suggests that the abundance of analytically useful ions in conventional ESI sources will be significantly enriched in the periphery region of the electrospray. Consequently, it sometimes has been the practice to sample electrosprays "off-axis" or orthogonal to the mass spectrometer inlet aperture, where the density of analytically useful ions is highest to obtain high sensitivity in ESI-MS applications. For conventionally used higher ESI flow rates, where initial droplet sizes are largest, this practice also reduces the introduction of residue droplets into the vacuum system that can subsequently contribute to contamination and detector noise. In the case of very low flow rate electrosprays, the initial droplet size is smaller and the problems due to droplet residue are much less, thus optimum performance is obtained by closer placement of the emitter to the sampling aperture. Since less time is needed for evaporation with such constrained ESI flow rates, problems associated with "on-axis" placement are minimized.

The simple theoretical model presented in this work incorporates several assumptions, some of which are based upon experimental measurements and observations. Several others have been made purely for the simplification of modeling itself, but done so as not to lose the meaning of the physical process involved. For example, in dealing with droplet evaporation, we make the assumption of an isolated droplet evaporating in a diffusion controlled regime and a constant environment. It is expected that the real droplet evaporation process for typical electrospray arrangements used in mass spectrometry will be much more complicated. For the submicron offspring droplets generated from fission process, both surface kinetics and Kelvin effect [19,20] will become increasingly important for droplet evaporation. Because the charged droplets move with respect to the gas phase environment in the electrospray, the convection effect will also play a role in the droplet evaporation process [20,21]. These effects will generally result in corrections to the droplet evaporation rate constant *K*, and consequently the evaporation time. However, because of the significant size difference between parent and offspring droplets, offspring droplets will still evaporate much faster than their parent droplet and become the primary source of

4. Conclusions

analyte ions in the ESI-MS process.

The theoretical analysis has allowed us to predict the size and the charge on both the progeny and the residue parent droplets, based on the possible number of progeny droplets, in each asymmetric fission process. In principle, the analysis can be carried on to each set of droplets, if there is no other competing droplet charge loss process existing in the electrospray, until charged ions are generated eventually. The calculated evaporation time required for both progeny droplets and residue parent droplet to become unstable after the initial fission process suggests that progeny droplets are the primary ion source in the ESI-MS, and that an important facet of ESI source efficiency will be the effectiveness of designs suitable for analyzing ions formed from different progeny generations (and greatly different times).

Acknowledgements

We thank the U.S. Department of Energy for financial support. Pacific Northwest National Laboratory is operated by Battelle Memorial Institute for the U.S. Department of Energy through contract no. DE-AC06-76RLO 1830.

References

- [1] A.G. Bailey, Electrostatic Spraying of Liquids, Wiley, New York, 1988.
- [2] M. Yamashita, J.B. Fenn, J. Phy. Chem. 88 (1984) 4451.
- [3] R.D. Smith, J.A. Loo, R.R. Ogorzalek Loo, M. Busman, H.R. Udseth, Mass Spectrom. Rev. 10 (1991) 359.
- [4] R.D. Smith, K. Tang, H.R. Udseth, A.L. Rockwood, Proceedings of 44th ASMS Conference on Mass Spectrometry and Allied Topics, Portland, OR, 1996, p. 1086.
- [5] S.A. Schaffer, K. Tang, G.A. Anderson, D.C. Prior, H.R. Udseth, R.D. Smith, Rapid Communications in Mass Spectrometry 11 (1997) 1813.
- [6] M. Dole, L.L. Mach, R.L. Hines, R.C. Mobley, L.P. Ferguson, M.B. Alice, J. Chem. Phys. 49 (1968) 2240.
- [7] J.V. Iribarne, B.A. Thomson, J. Chem. Phys. 64 (1976) 2287.
- [8] A.L. Burlingame, R.K. Boyd, S.J. Gaskell, Anal. Chem. 70(1998) 647R–716R.
- [9] L. Rayleigh, Philos. Mag. 14 (1882) 184.
- [10] G.A. Dawson, J. Geophys. Res. 78 (1973) 6364.
- [11] G. Chen, A. Gomez, 24th Symposium (International) on Combustion, Pittsburgh, PA, 1992, pp. 1531–1539.
- [12] A. Doyle, D.R. Moffet, B. Vonnegut, J. Colloid. Sci. 19 (1964) 136.
- [13] M.A. Abbas, J. Latham, J. Fluid Mech. 30 (1967) 663.
- [14] D.C. Taflin, T. L. Ward, E. J. Davis, Langmuir 5 (1989) 376.
- [15] A. Gomez, K. Tang, Phys. Fluids 6 (1994) 404.
- [16] P. Kebarle, L. Tang, Anal. Chem. 65 (1993) 927A.
- [17] D.E. Rosner, Transport Processes in Chemically Reacting Flow Systems, Butterworth, Boston, 1986.
- [18] K. Tang, A. Gomez, Phys. Fluids 6 (1994) 2317.
- [19] C.N. Davis, in D.T. Shaw (Ed.), Fundamentals of Aerosol Science, Wiley, New York, 1978.
- [20] N.A. Fuchs, Evaporation and Droplet Growth in Gaseous Media, Pergamon, London, 1959.
- [21] C.K. Law, Prog. Energy Combust. Sci. 8 (1982) 171.